# Computer Graphics

# - Volume Rendering -

Pascal Grittmann, Jaroslav Křivánek

Using pictures from: **Monte Carlo Methods for Physically Based Volume Rendering**; SIGGRAPH 2018 Course; Jan Novák, Iliyan Georgiev, Johannes Hanika, Jaroslav Křivánek, Wojciech Jarosz

# Overview

- So far:
  - Light interactions with surfaces
  - Assume vacuum in and around objects
- This lecture:
  - Participating media
  - How to represent volumetric data
  - How to compute volumetric lighting effects
  - How to implement a very basic volume renderer

# Fog & clouds

Steve Lacey

# Underwater

source: dailypictures.info

# Surface or volume?

source: Flickr

source: Studio Lernert & Sander

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**Arrival**. Copyright © 2016 Paramount Pictures

**Big Hero 6**. Copyright © 2014 Walt Disney Enterprises, Inc.

**Mortal Engines**. Copyright © 2018 Universal Studios

# Fundamentals

# Volumetric Effects

- Light interacts not only with surfaces but everywhere inside!
- Volumes scatter, emit, or absorb light

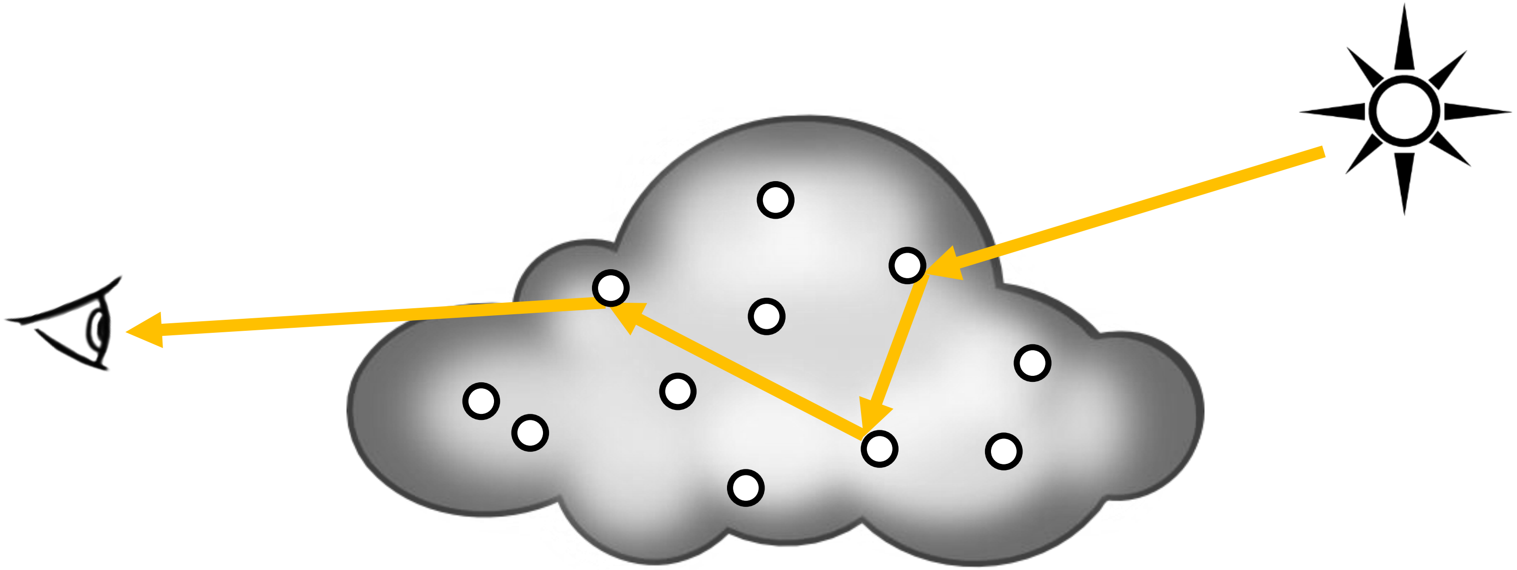
http://coclouds.com

http://wikipedia.org

http://commons.wikimedia.org

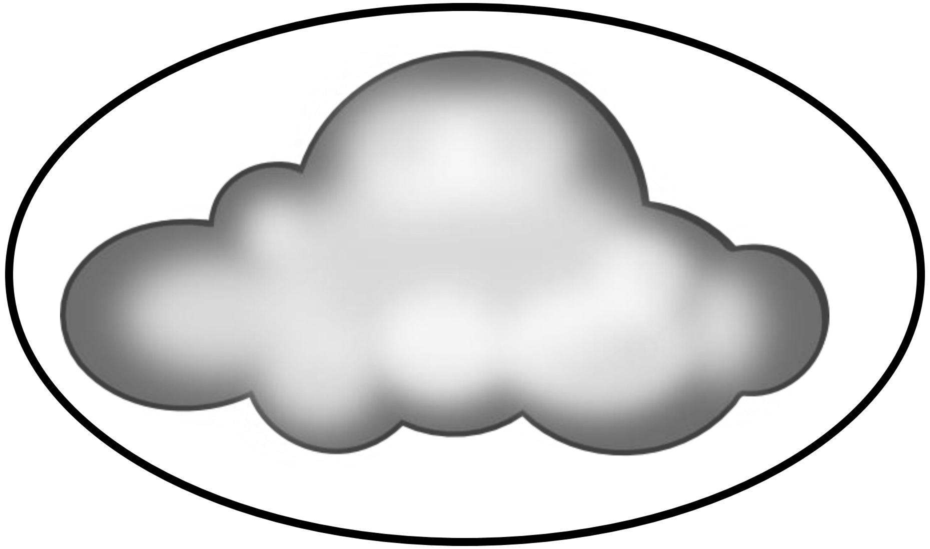
# Approximation: Model Particle Density

- Modeling individual particles of a volume is, of course, not practical
- Instead, represent statistically using the average density
- (Same idea as, e.g., microfacet BSDFs)



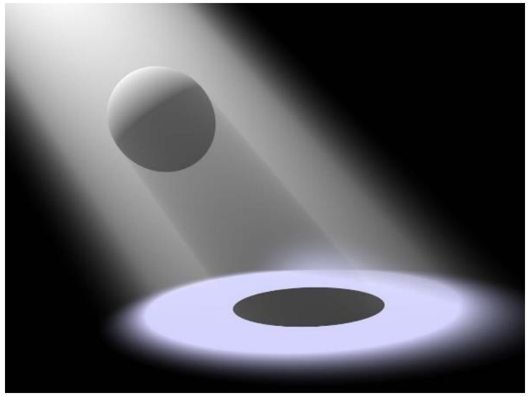
# Volume Representation

- Many possibilities (particles, voxel octrees, procedural,…)
- A common approach: Scene objects can “contain” a volume



# Volume Representation

- Homogeneous:
  - Constant density
  - Constant absorption, scattering, emission,
  - Constant phase function (later)



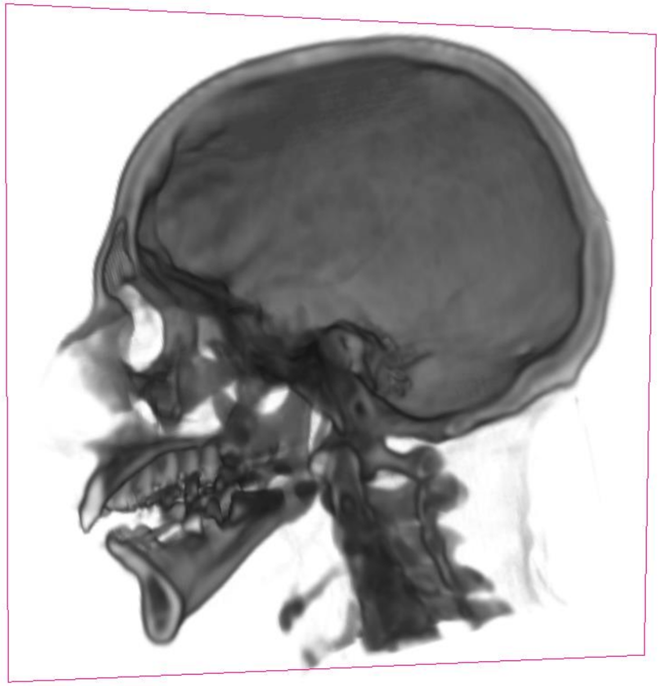
- Heterogeneous:
  - Coefficients and/or phase function vary across the volume
  - Can be represented using **3D textures**
  - (e.g., voxel grid, procedural)

http://wikipedia.org

# Data Acquisition

- Real-world measurements via tomography
- Simulation, e.g.,
  - Fluids,
  - Fire and smoke,
  - Fog

https://docs.blender.org

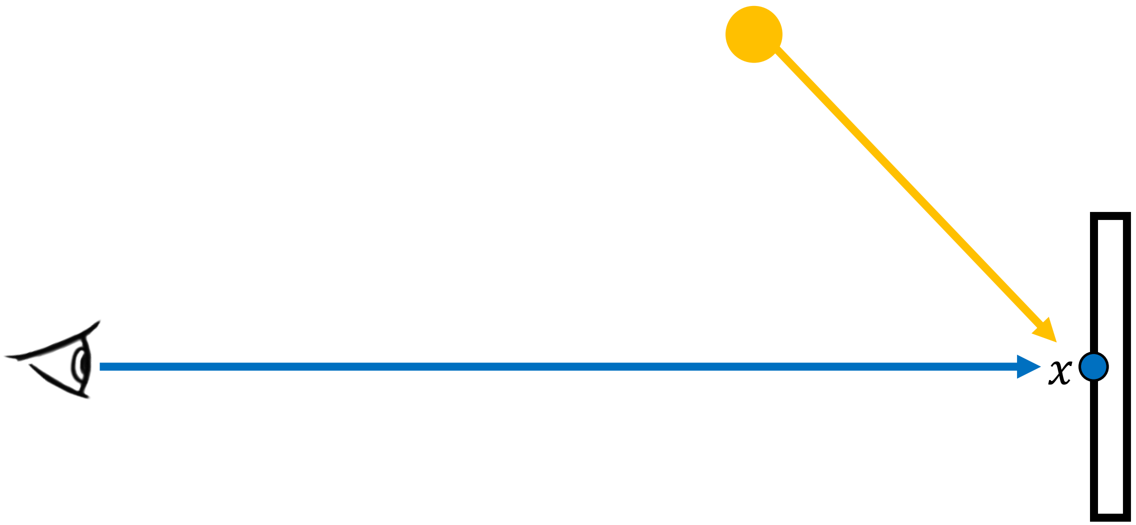


# Simulating Volumes

Mathematical Formulation of Volumetric Light Transport

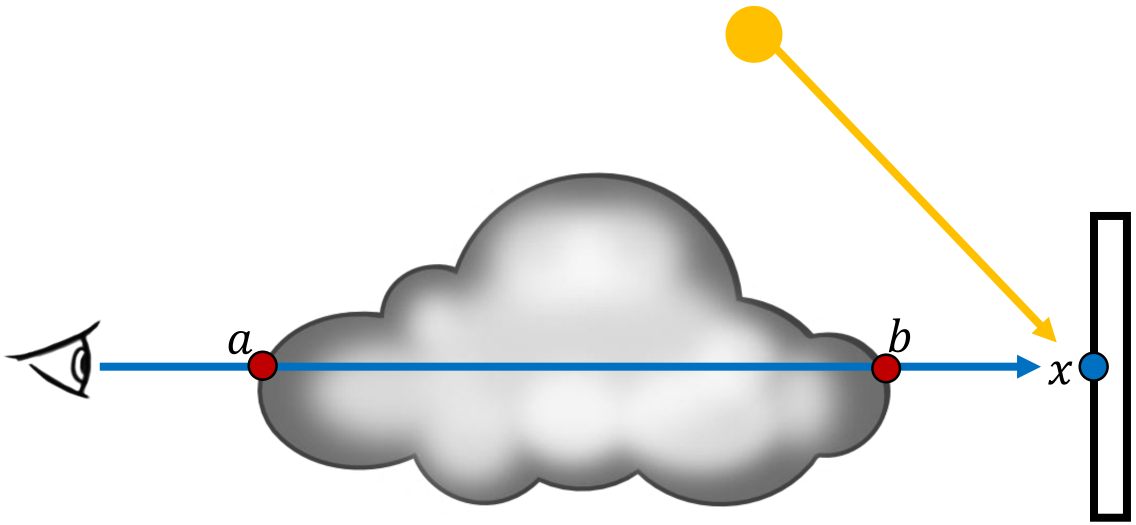
# So far: Assume Vacuum

- Compute $L_o(x, \omega_o)$ using the rendering equation



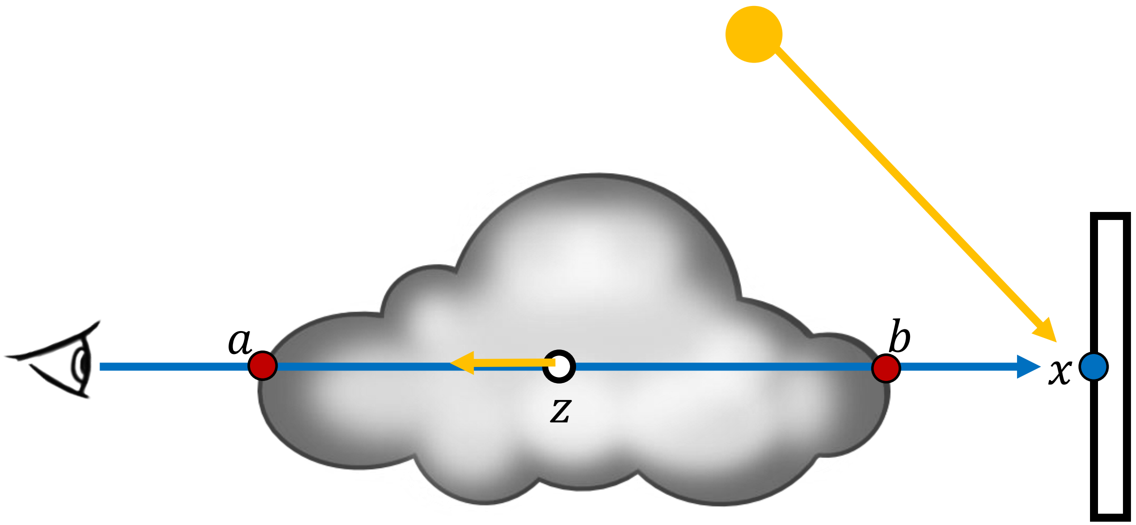
# Volume Absorbs and Scatters Light

- Compute $L_o(x, \omega_o)$ using the rendering equation
- Only a fraction $T(a, b)L_o(x, \omega_o)$ arrives at the eye



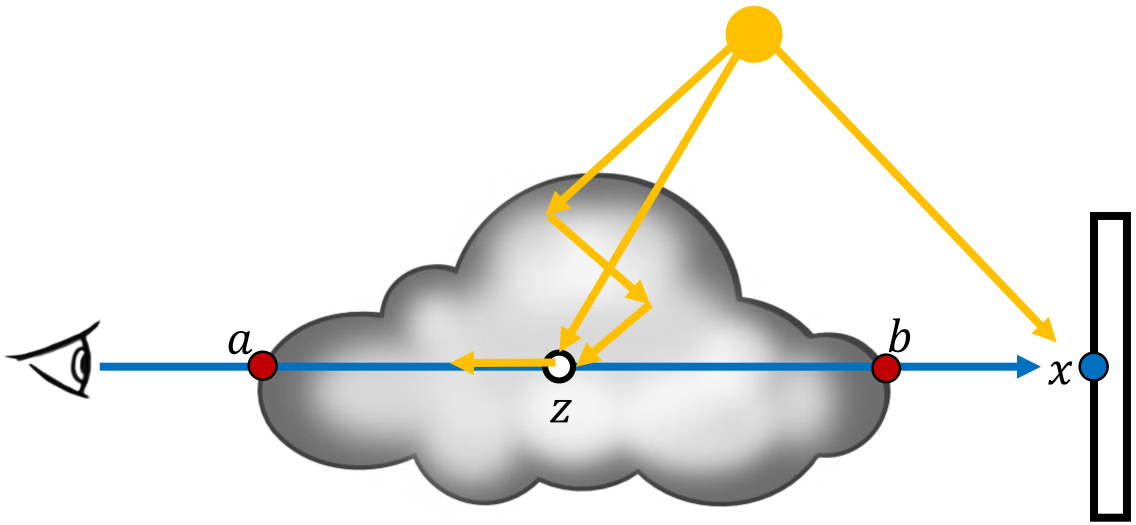
# Volume Emits Light

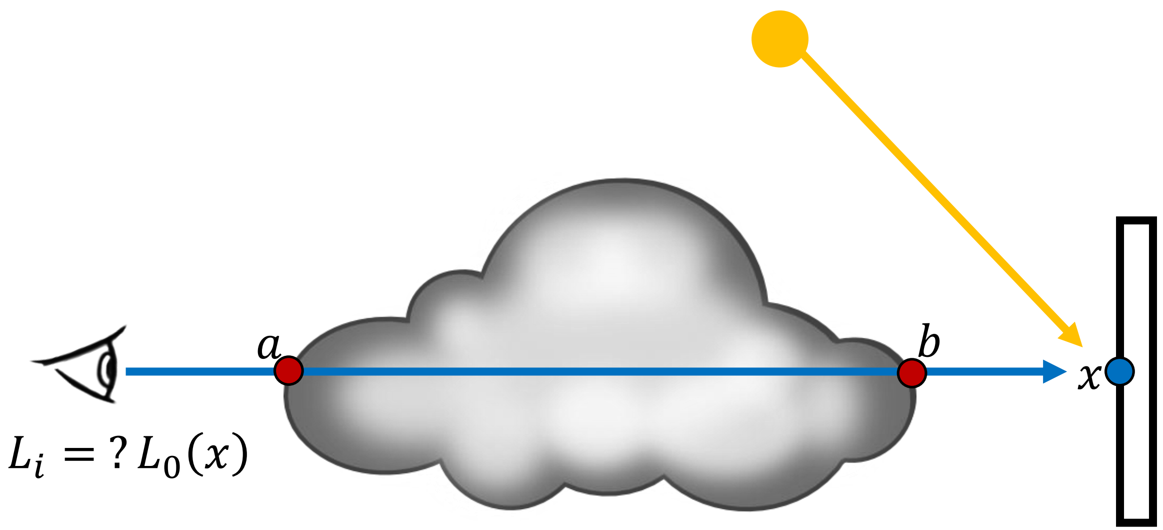
- Compute $L_o(x, \omega_o)$ using the rendering equation
- Only a fraction $T(a, b)L_o(x, \omega_o)$ arrives at the eye
- Every point $z$ between $a$ and $b$ might emit light



# Volume Scatters Light

- Compute $L_o(x, \omega_o)$ using the rendering equation
- Only a fraction $T(a, b)L_o(x, \omega_o)$ arrives at the eye
- Every point $z$ between $a$ and $b$ might emit light
- Every point $z$ might be illuminated through the volume







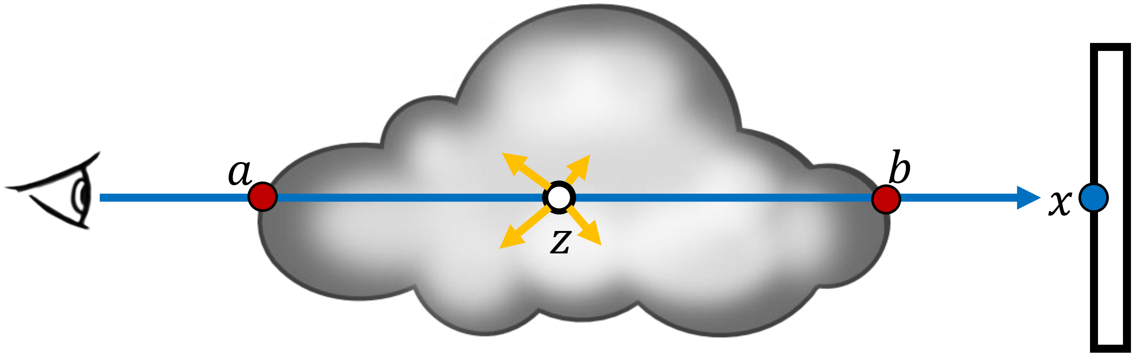
http://commons.wikimedia.org

# Attenuation

Computing Absorption and Out-Scattering

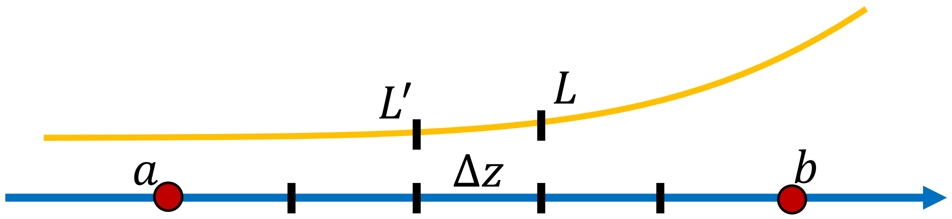
# Attenuation = Absorption + Out-Scattering

- Every point in the volume might absorb light or scatter it in other directions
- Modeled by absorption and scattering coefficients: $\mu_a(z)$ and $\mu_s(z)$ (both in $[\mathrm{m}^{-1}]$)
- Might depend on position, direction, time, wavelength,…
- For simplicity: we assume only positional dependence



# Computing Absorption – Intuition

- Consider a small segment $\Delta z$
- Along that segment, radiance is reduced from $L$ to $L'$
- $L' = L - L(\mu_a \Delta z)$
- Where $\mu_a$ is the percentage of radiance that is absorbed (per unit distance)



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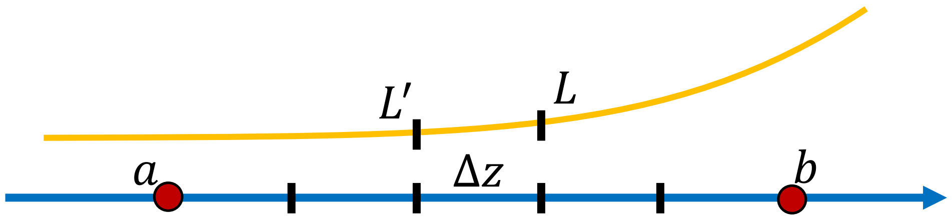
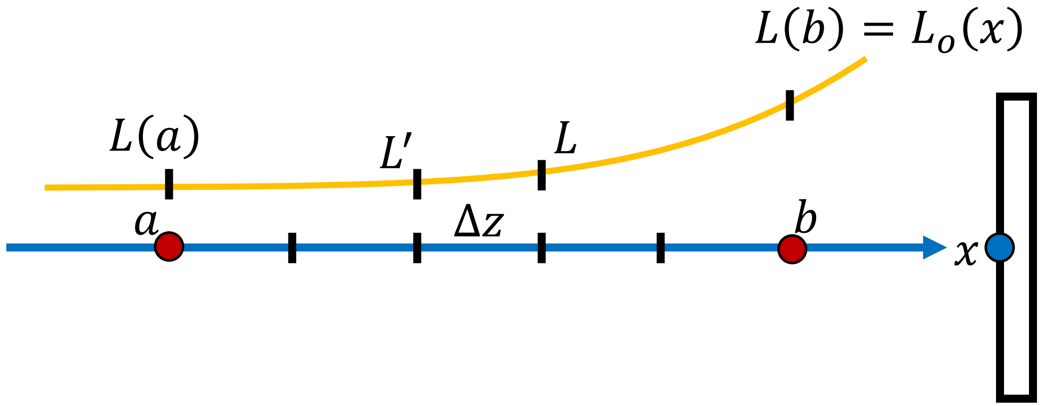
- Lets rewrite this:
- $\Delta L = L' - L = -L\mu_a\Delta z$



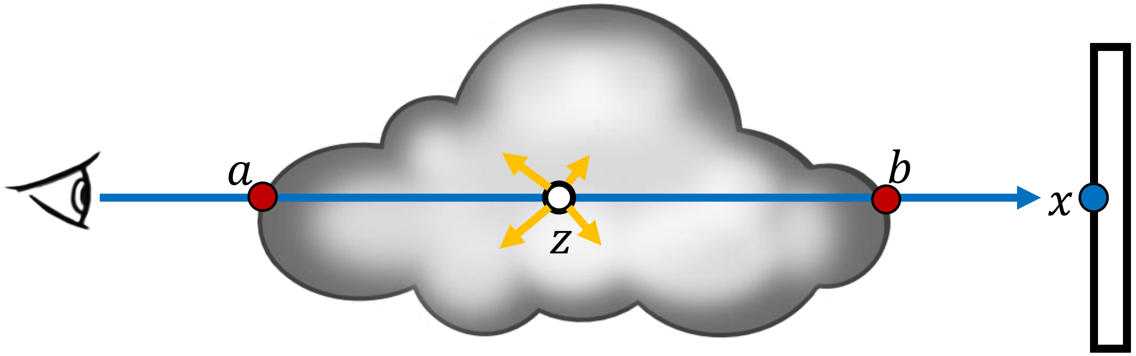
# Computing Absorption – Exponential Decay

- $\Delta L = -\mu_a \, L \, \Delta z$
- For infinitely small $\Delta z$, this becomes
- $dL = -\mu_a \, L \, dz$
- A differential equation that models exponential decay!
- Solution: $L(a) = L_o(x) \, e^{-\int_0^z \mu_a(t) \, dt}$



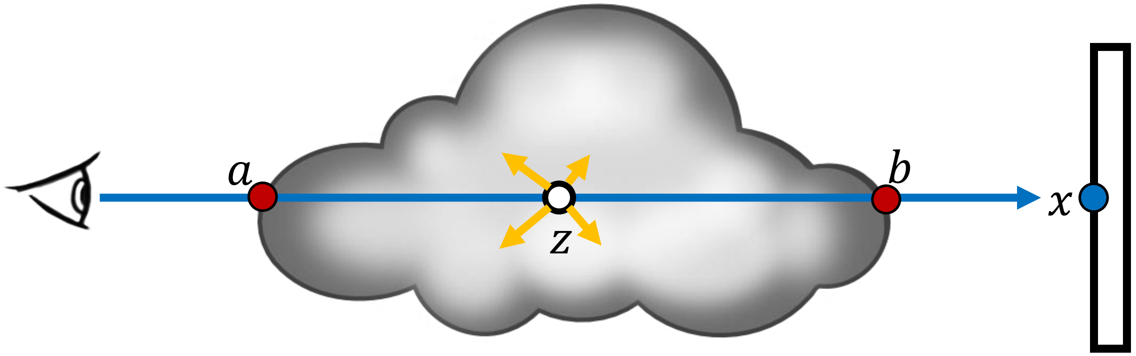
# Computing Out-Scattering

- Same as absorption, only different factor!
- $L(a) = L_o(x)\, e^{-\int_0^z \mu_s(t)\, dt}$



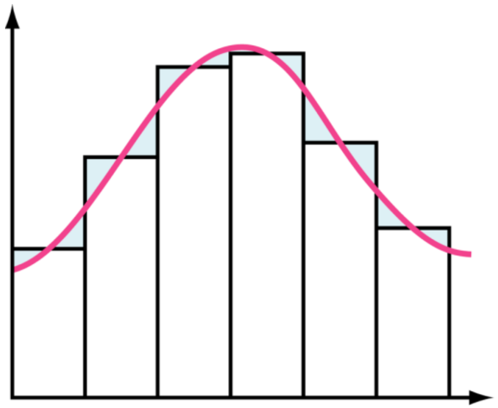
# Computing Attenuation

- Fraction of light that is either absorbed or out-scattered (per unit distance)
- $\mu_t = \mu_a + \mu_s$
- Many different names: extinction / attenuation / transport coefficient
- $L(a) = L_o(x)\, e^{-\int_0^z (\mu_a(t)+\mu_s(t))\, dt}$
- Attenuation: $T(a,b) = e^{-\int_b^a \mu_t(t)\, dt}$



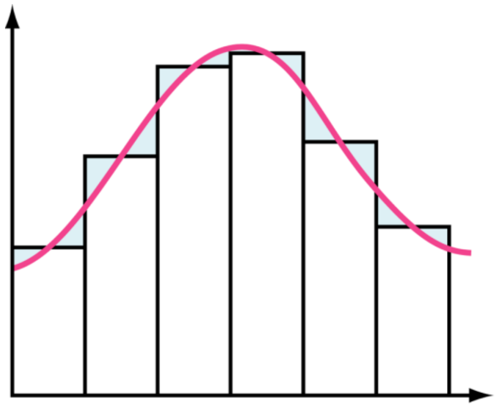
# Estimating Attenuation

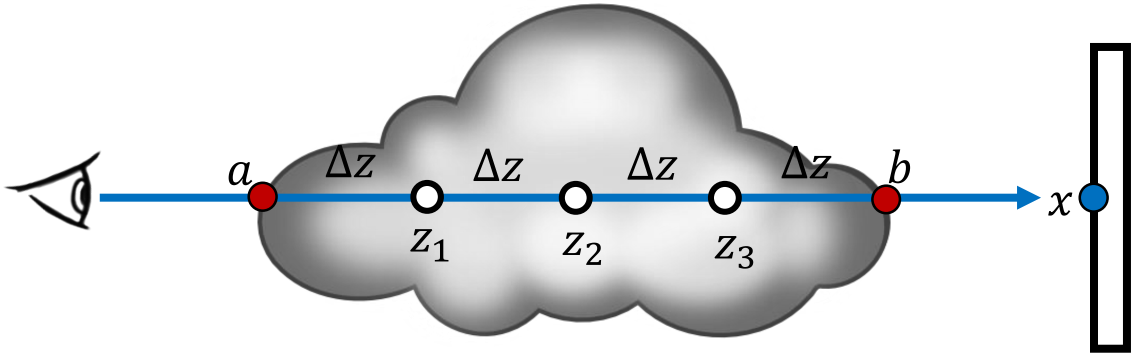
- We need to evaluate another integral:
  - $T(a,b) = e^{-\int_b^a \mu_t(t)\,dt}$
- Many approaches, e.g., ~~Monte Carlo integration~~ or **deterministic quadrature**

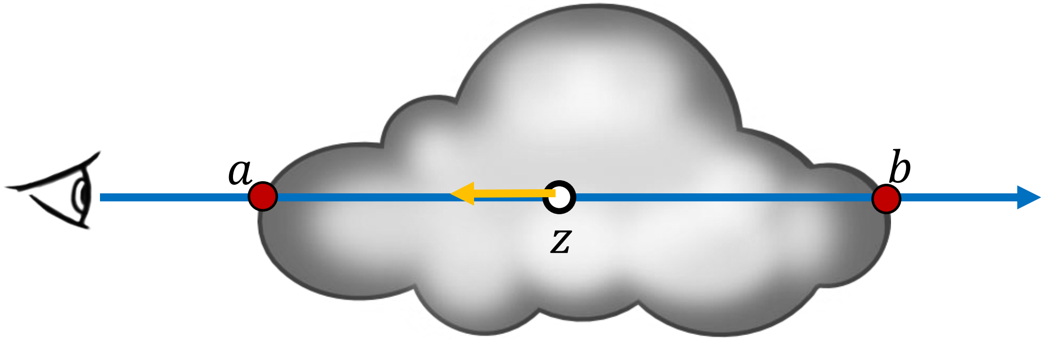


# Estimating Attenuation – Ray Marching

- We need to evaluate another integral:
  - $T(a,b) = e^{-\int_b^a \mu_t(t)\, dt}$
- Many approaches, e.g., ~~Monte Carlo integration~~ or **deterministic quadrature**
- Ray marching: evaluate at discrete positions (fixed stepsize $\Delta z$)
- $\int_b^a \mu_t(t)\, dt \approx \sum_i \mu_t(z_i + \varepsilon)\, \Delta z$
- Randomized **offset $\boldsymbol{\varepsilon}$** for each ray to avoid aliasing problems







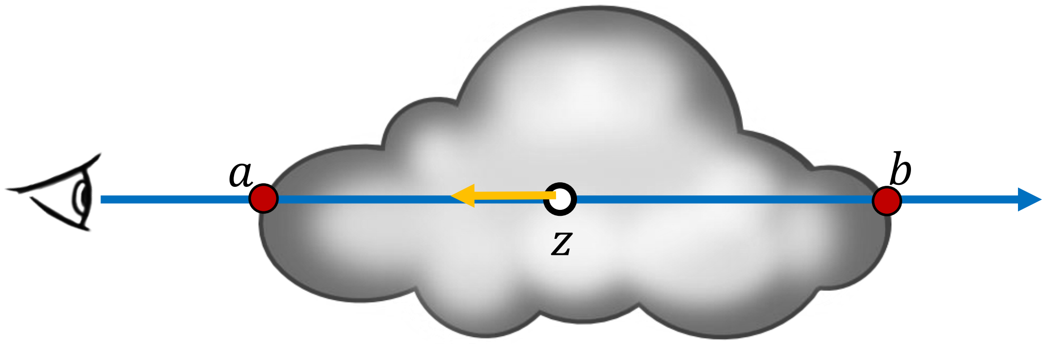

# Emission

Explosions!

http://wikipedia.org

# Every Point Might Emit Light

- Assume $z$ emits $L_e(z)$ towards $a$
- Some of that light might be absorbed or out-scattered: It is attenuated
- $L(a) = L_e(z)\, T(z, a)$



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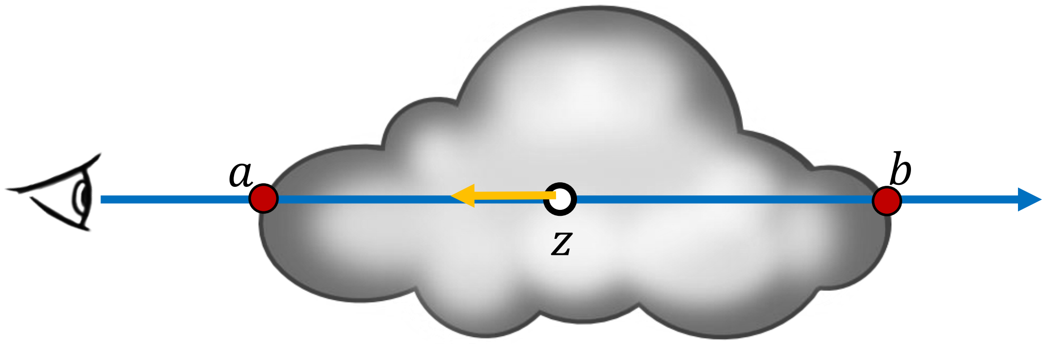
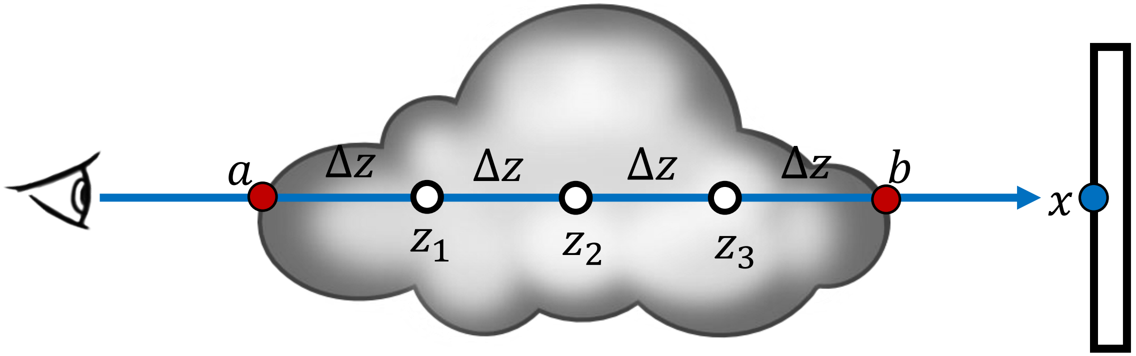
- Happens at every point along the ray!
- $L(a) = \int_a^b L_e(z)\, T(z, a)\, dz$

- Another integral…



# Ray Marching for Emission

- Same as before: integrate via quadrature
- $\int_a^b L_e(z)\, T(z,a)\, dz \approx \sum_i L_e(z_i)\, T(z_i,a)\, \Delta z$
- Attenuation $T(z_i, a)$ estimated as before



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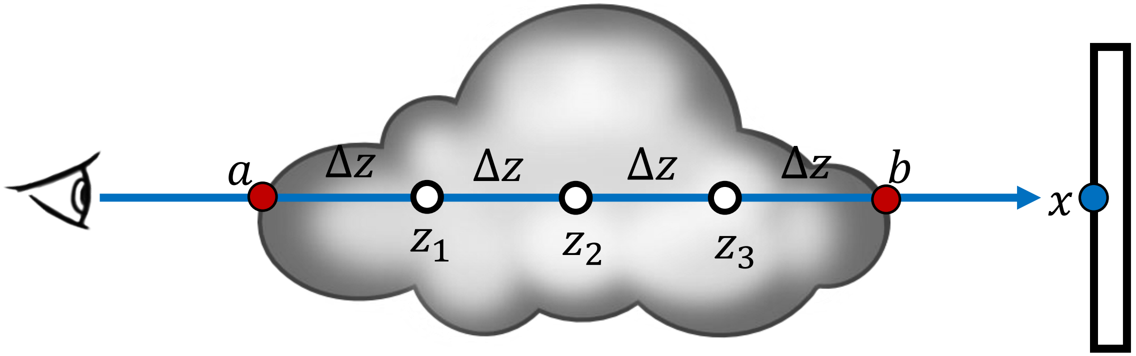
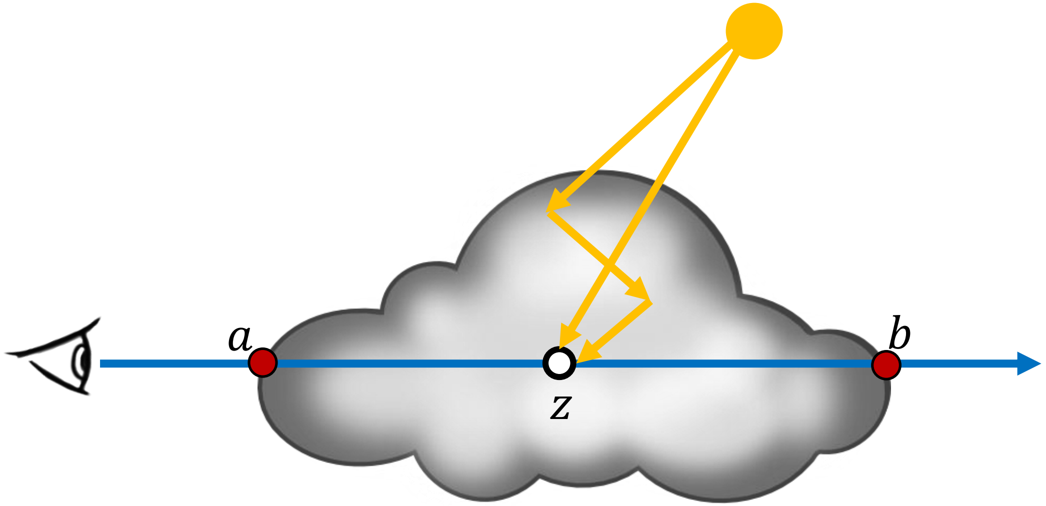
- Attenuation can be incrementally updated:
    - $T(z_i,a) = T(z_{i-1},a)\, T(z_i, z_{i-1})$
    - (because it is an exponential function)





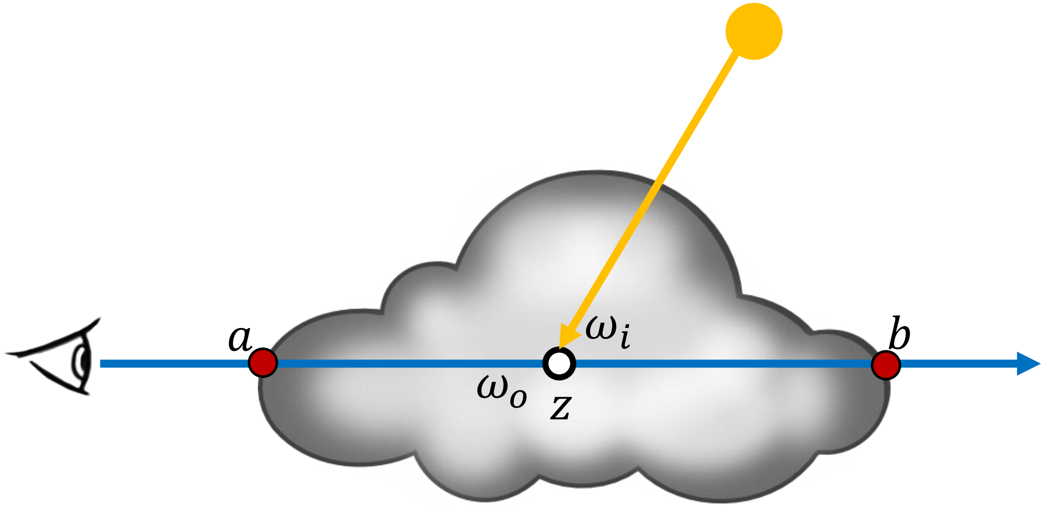


# In-Scattering

Accounting for “Reflections” Inside the Volume

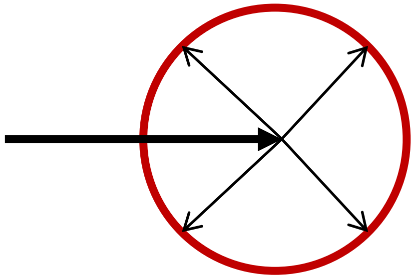
# Direct Illumination (Single-scattering)

- Account for the (attenuated) direct illumination at every point $z$
- Similar to the rendering equation:
- $L_o(z, \omega_o) = \int_\Omega L_i(x, \omega_i)\, f_p(\omega_i, \omega_o)\, d\omega_i$
- Integration over the whole sphere $\Omega$
- The **phase function** $f_p$ takes on the role of the BSDF



# Phase Functions

- $f_p(\omega_i, \omega_o)$
- Describe what fraction of light is reflected from $\omega_i$ to $\omega_o$
- Similar to BSDF for surface scattering
- Simplest example: isotropic phase function
  - $f_p(\omega_i, \omega_o) = \frac{1}{4\pi}$
  - (energy conservation: $\int_\Omega \frac{1}{4\pi} d\omega = 1$)



# Phase Functions: Henyey-Greenstein

- Widely used
- Easy to fit to measured data

- $f_p(\omega_i, \omega_o) = \frac{1}{4\pi} \frac{1-g^2}{(1+g^2+2g\cos(\omega_i,\omega_o))^{\frac{3}{2}}}$

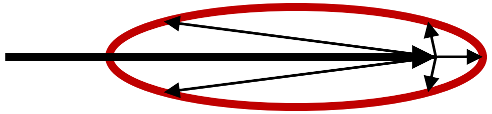
- $g$: asymmetry (scalar)
- $\cos(\omega_i, \omega_o)$: cosine of angle between the directions

# Henyey-Greenstein: Asymmetry Parameter

- $g = 0$: isotropic
- Negative $g$: back scattering
- Positive $g$: forward scattering

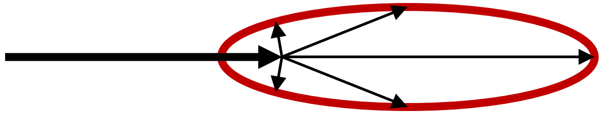
Back Scattering

http://commons.wikimedia.org



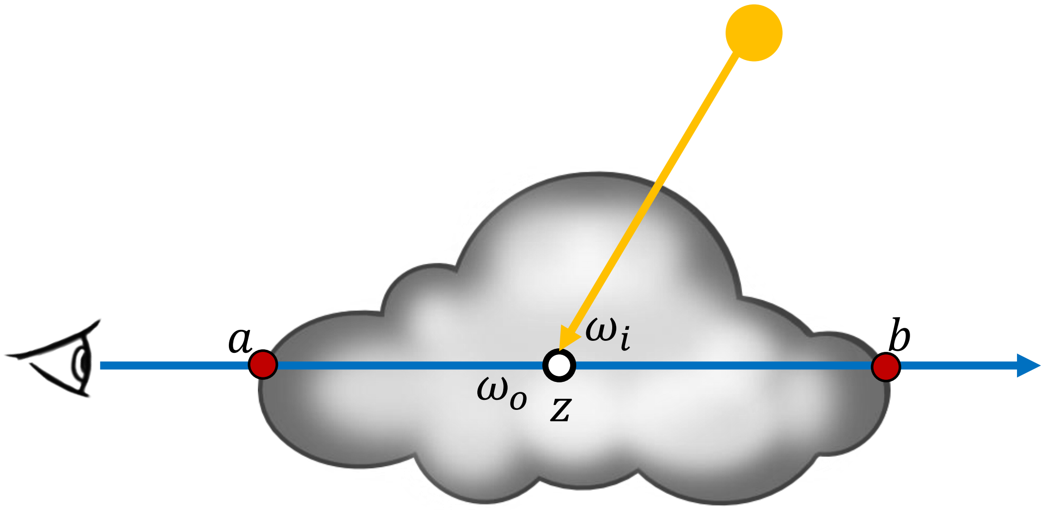
Forward Scattering

http://coclouds.com



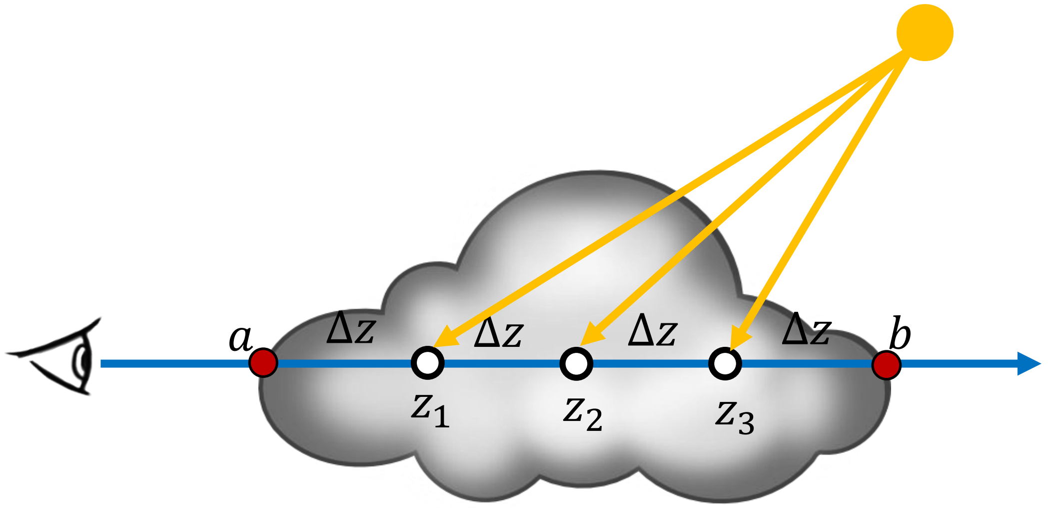
# How to Estimate Volumetric Direct Illumination

- Reflected radiance at a point $z$: $L_o(z, \omega_o) = \int_\Omega L_i(x, \omega_i)\, f_p(\omega_i, \omega_o)\, d\omega_i$
- In our framework:
  - Sum over all point lights (as for surfaces)
  - Trace shadow ray (as for surfaces)
  - Estimate attenuation along the shadow ray (as for surfaces)



# Ray Marching to Compute In-Scattering

- Same as for emission
- Goal: estimate the integral $\int_a^b T(z,a)\, L_i(z)\, f_p\, dz$
- Quadrature:
  - $\int_a^b T(z,a)\, L_i(z)\, f_p\, dz \approx \sum_i T(z_i,a)\, L_i(z_i)\, f_p\, \Delta z$



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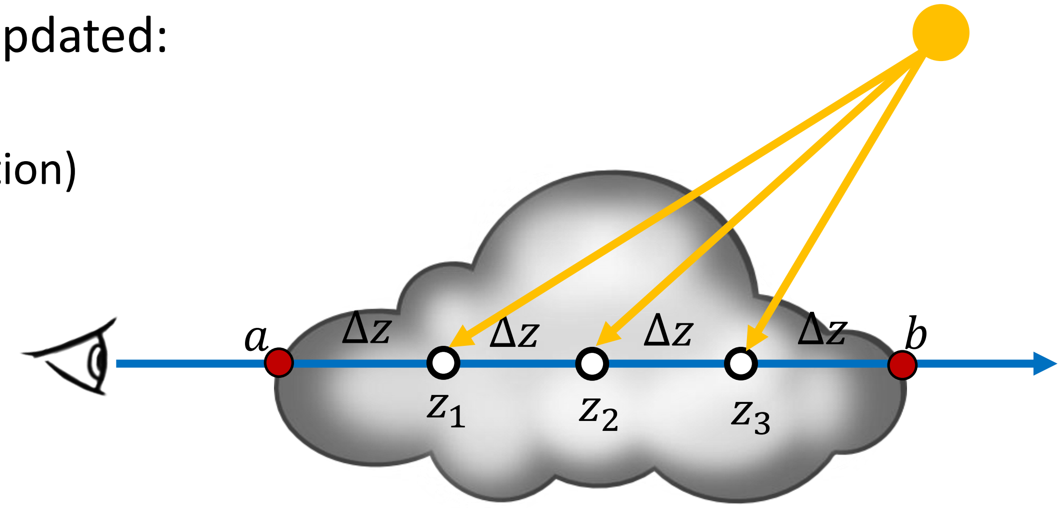
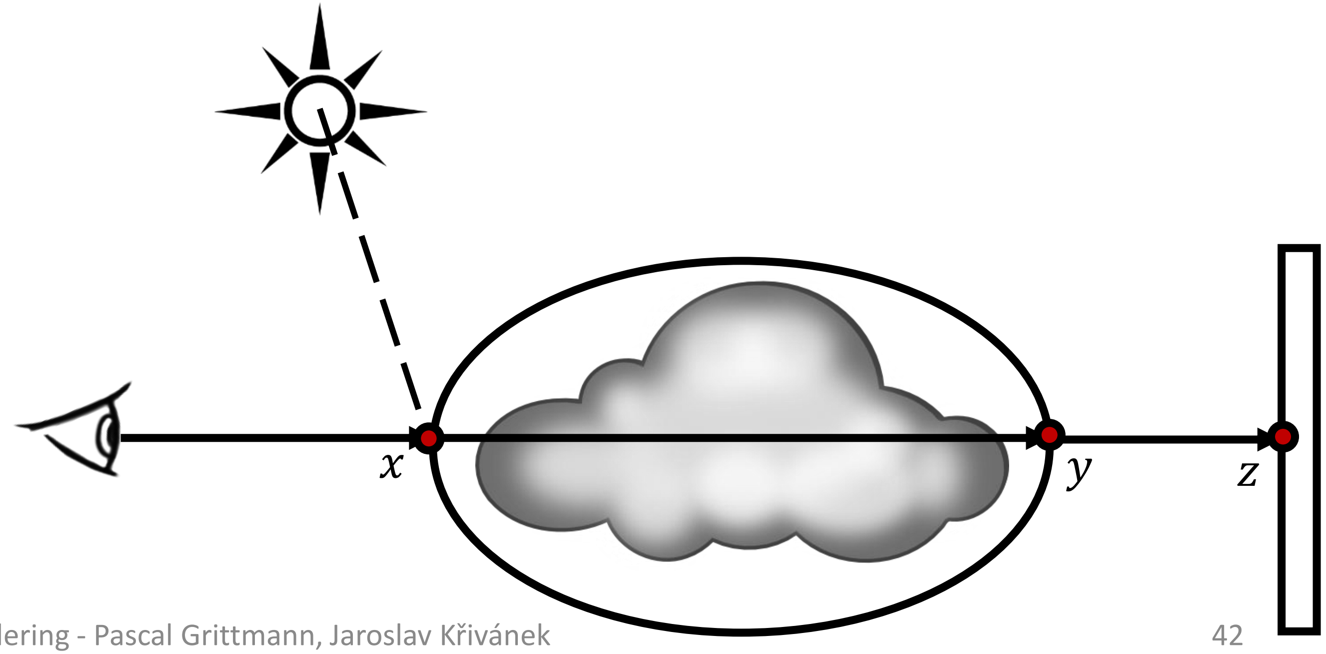


# Putting it all Together

A Simple Volume Integrator

# A Simple Volume Integrator

- Estimate direct illumination at x (as before)
- If volume: continue straight ahead until no volume (yields intersections y, z)



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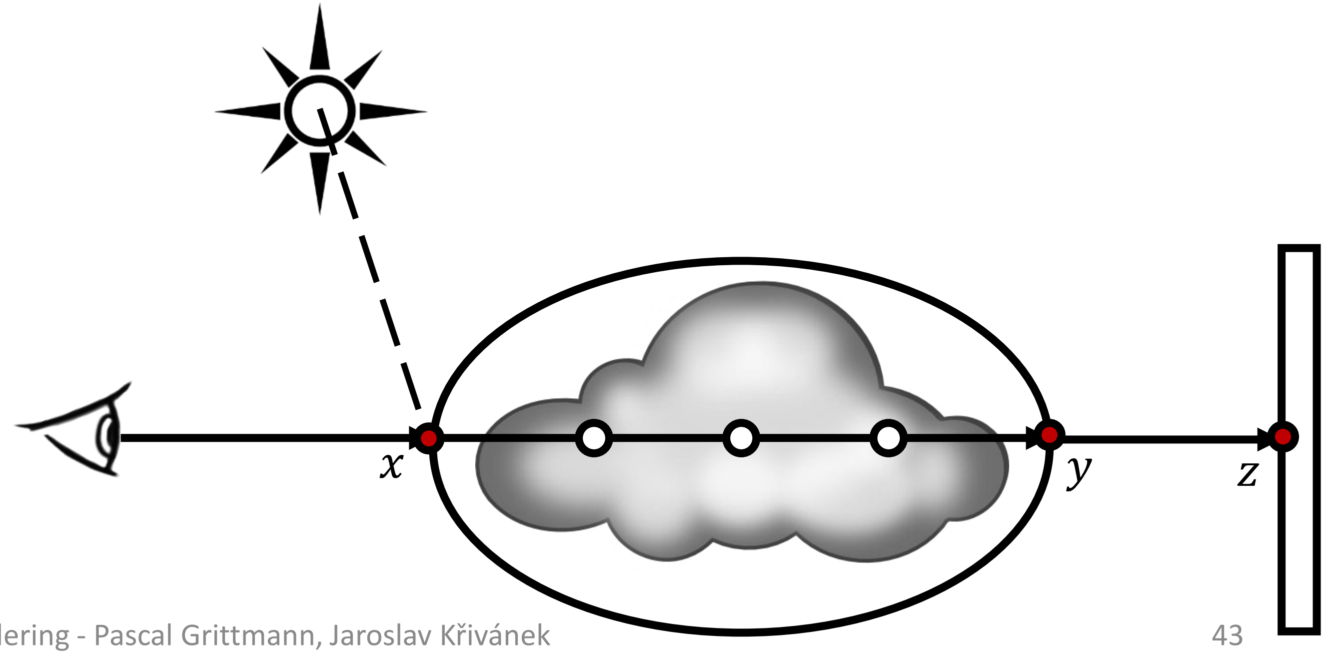
- Ray marching to estimate attenuation, emission,



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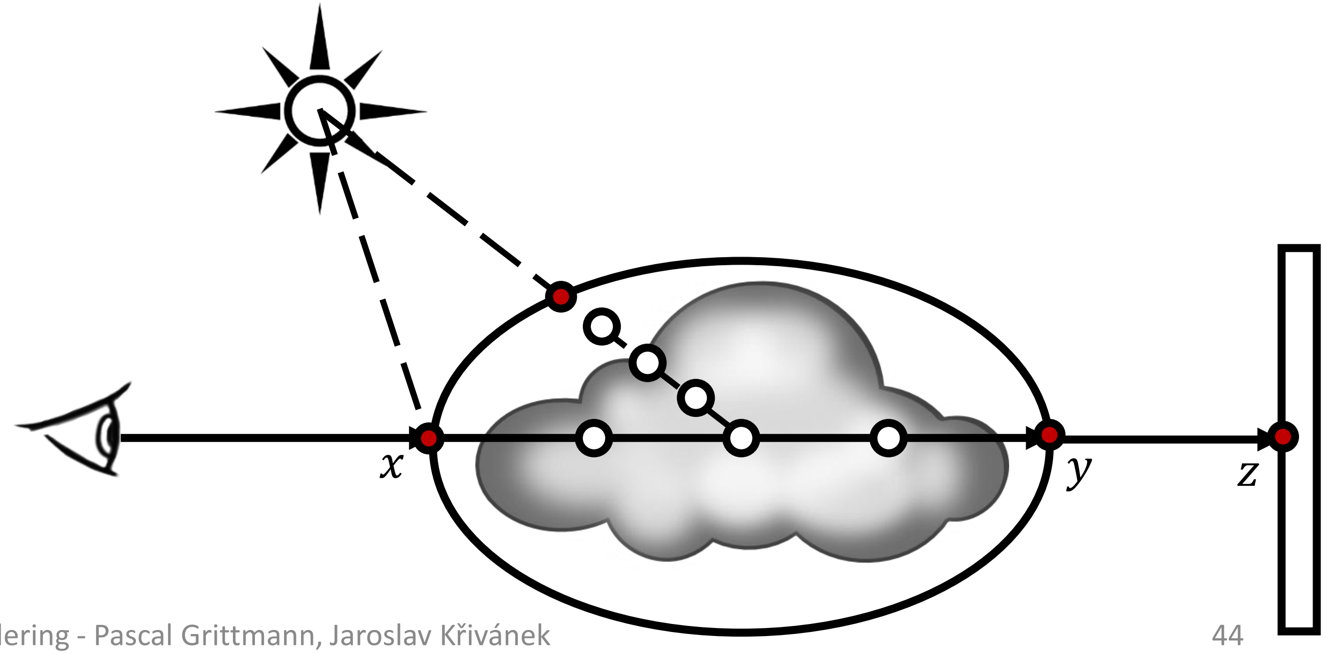

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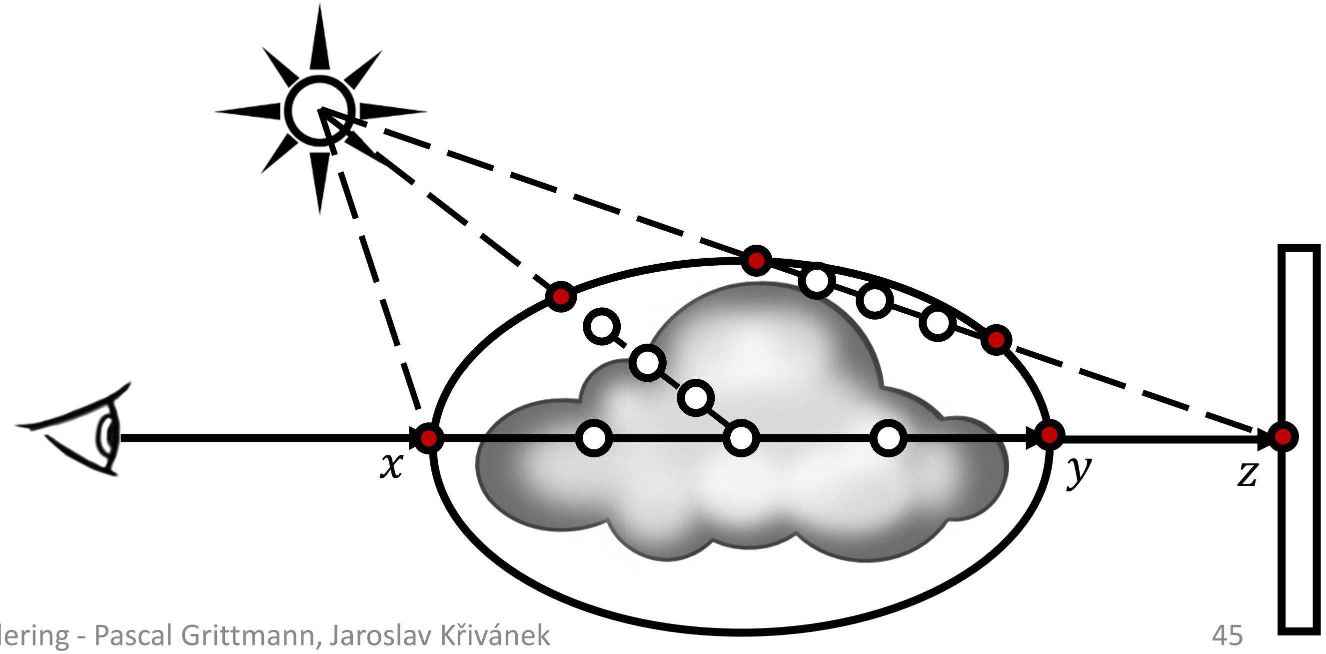
- Compute illumination at z (as before)



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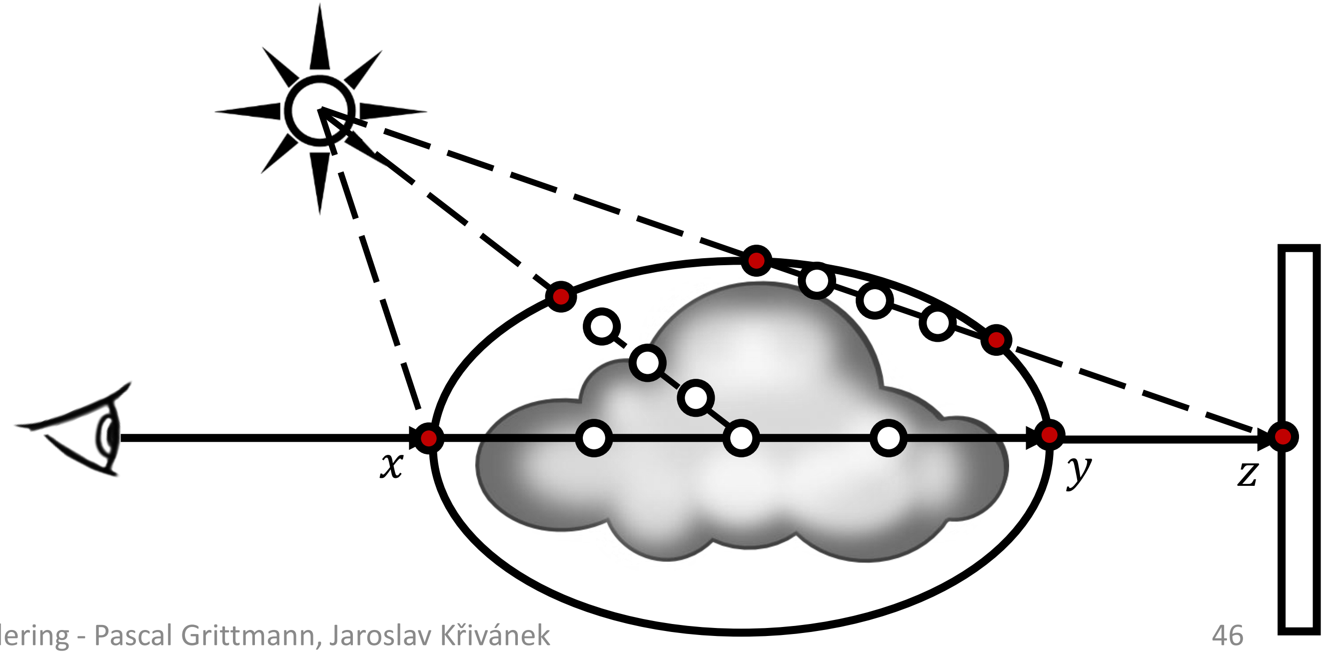
- Compute illumination at z (as before)
- Add together:
  - Attenuated illumination from z
  - Volumetric emission along $\overline{xy}$
  - In-scattering along $\overline{xy}$
  - Direct illumination at $x$



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